Math 180 Exam 1 Review

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1. An object is launched into the air. Its position at time t is given by $s(t) = -5t^2 + 5t + 10$.

   (a) Find the average velocity on the interval $[1, 2]$.

   (b) Find the average velocity on the interval $[0, h]$. Simplify your answer.

   (c) Find the instantaneous velocity at $t = 0$ using the definition of the derivative.

   (d) Write the equation of the tangent line to the graph of $s(t)$ through the point $(0, 10)$.

2. (Briggs and Cochran, Exercise 2.2.7) Use the supplied graph of $h(x)$ to determine the following.

   (a) $h(2)$

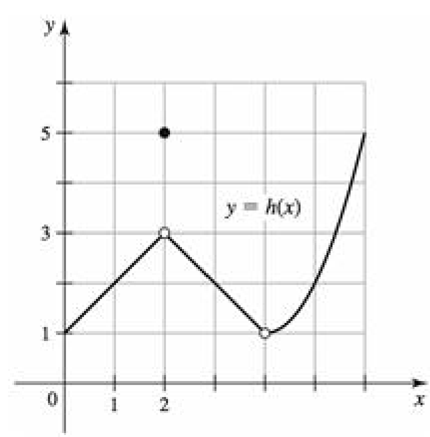
   (b) $\lim_{x \to 2} h(x)$

   (c) $h(4)$

   (d) $\lim_{x \to 4} h(x)$

   (e) Is h continuous at $x = 2$?

   (f) How about at $x = 4$?



3. Calculate each of the following limits if it exists.

(a) $\lim_{x \to 5} \frac{x-5}{x^2-4x-5}$

(b) $\lim_{x \to -1^+} \frac{x-5}{x^2-4x-5}$

(c) $\lim_{x \to 0} \frac{2x}{\tan(3x)}$

4. Consider the function $f(x) = \frac{\sqrt{3x^2+1}-2}{1-x}$. Calculate the following:

(a) $\lim_{x \to -\infty} f(x)$

(b) $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to 1} f(x)$

(d) $\lim_{x \to \infty} f(x)$

5. Prove, using the formal definition of limits, that $\lim_{x \to 1} 2x - 3 = -1$.

6. Calculate the derivatives of the following functions:

(a) $\dfrac{5}{1 + e^{-3t}}$

(b) $\dfrac{1}{\sqrt{\pi}} e^{-x^2}$

(c) $x^5 \sin(x) + x^4 \cos(x) + 1$

(d) $\sqrt{1 + \sin^2(x)}$

7. On which intervals are the following functions continuous?

   (a) $\dfrac{x}{x^2+1}$

   (b) $\sqrt{3x-1}$

   (c) $\dfrac{x+1}{x^2+3x+2}$

8. A piñata hangs from a spring. Its height at time t given by $y(t) = 15\sin(2\pi t) + 20$.

   (a) Find the velocity and acceleration functions for the piñata.

   (b) What are the height, velocity, and acceleration of the piñata at $t = 3$?

9. Consider the function $f(t) = t^4 - 5t^3 + 2$.

   (a) Show that f has a root between $t = 0$ and $t = 1$. Justify your answer.

   (b) Improve upon this result by finding an interval of length $\frac{1}{4}$ on which f has a root. (Remember – no calculators!)